THEORETICAL AND EXPERIMENTAL INVESTIGATION OF A THREE-DIMENSIONAL MAGNETIC-SUSPENSION BALANCE FOR DYNAMIC-STABILITY RESEARCH IN WIND TUNNELS

Semiannual Status Report
May 31, 1965 to February 28, 1966

National Aeronautics and Space Administration
Grant NGR 47-005-029

H. M. Parker
Principal Investigator

RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA

Report No. AST-4030-102-66U
March 1966

Copy No. 3

[redacted]

A series of basic magnetic field calculations have been made and design charts have been prepared which enable designs of $\tan^{-1}\sqrt{2}$ and $\tan^{-1}\sqrt{8}$ magnetic balances as well as drag augmented $\tan^{-1}\sqrt{8}$ systems. While the coil cross sections are optimized in a certain sense and as a result may not be suitable for actual coil shapes, they are considered to be close enough to practical shapes for design purposes. The basic geometries have been worked out and while at first sight the geometrical relationships seem to be complicated, experience has shown that one quite quickly is able to visualize them.

It is appropriate, perhaps, to remind the reader of the principles on which the UVA magnetic balance operates. A small sphere of magnetic material is placed at the support point and is uniformly magnetized in the tunnel axis direction by the main magnetic field produced by the main field coils. A pair of gradient coils with a common axis, symmetrically placed on either side of the support point, and carrying opposing currents produce a field gradient (but no field) at the support point. If the angle between the axis of the gradient coils and the tunnel axis (direction of magnetization of the sphere) is zero, the direction of the force on the sphere is parallel to the tunnel axis (z-force gradient coils); if the angle is $\tan^{-1}\sqrt{2}$, the force is perpendicular to the tunnel axis in the plane containing the tunnel axis and the gradient coil axis ($\tan^{-1}\sqrt{2}$ coil); if the angle is $\tan^{-1}\sqrt{8}$, the force direction is in the tunnel axis-gradient coil axis plane and makes an angle $\tan^{-1}\frac{1}{\sqrt{2}}$ with the tunnel axis ($\tan^{-1}\sqrt{8}$ coil).

A $\tan^{-1}\sqrt{2}$ balance system consists of two zero angle coils, z-force coils, to produce a force parallel to the tunnel axis, two sets of $\tan^{-1}\sqrt{2}$ coils to produce two forces perpendicular to each other and perpendicular to the tunnel axis, and two main field coils to produce the magnetizing field. A typical configuration is shown in Figure 5. (Here and in Figure 6 only one quadrant of the section is shown.)

A $\tan^{-1}\sqrt{8}$ balance system consists of three pairs of gradient coils with the three axes at $\tan^{-1}\sqrt{8}$ with the tunnel axis and space 120° around it, and a pair of main field coils. The force directions lie along the edges of a cube, the major diagonal of which coincides with the tunnel axis. The drag augmented $\tan^{-1}\sqrt{8}$ system additionally has a pair of z-force coils (zero angle) to produce an extra force parallel to the drag (parallel to the tunnel axis). A typical configuration is shown in Figure 6.

In designing a magnetic balance, it is obvious that space to place coils to produce magnetic fields or magnetic field gradients at the support point (or symmetry point) is at a premium since the process rapidly becomes less efficient as the current carrying coils are moved further away. Nevertheless, in the design charts which have been prepared, two (at least) rather arbitrary and less than optimum choices have been made. First, there is space in the $\tan^{-1}\sqrt{2}$ system to add additional z-force gradient coils between the tunnel wall and the $\tan^{-1}\sqrt{2}$ coils close to the symmetry plane. It is believed that the addition of such coils will not change the general conclusion that the $\tan^{-1}\sqrt{2}$ system is inferior to the $\tan^{-1}\sqrt{8}$ system but the point will be verified. Second, it was arbitrarily decided that the inner surface of the main field coils should be cylindrical even though there is space to build them in to smaller radii at positions removed axially from the symmetry plane. This choice is not expected to produce a severe penalty and it does contribute to a desirable ease of assembly.

BASIC GEOMETRICAL CONSIDERATIONS

Tunnel Wall Contact Line

Obviously the coil system must remain outside the tunnel wall. Therefore, the inner limit of the $\tan^{-1}\sqrt{2}$ and $\tan^{-1}\sqrt{8}$ gradient coils is a conical shape determined by the line contact with the (cylindrical) tunnel wall. That contact line lies in the gradient axis-tunnel axis plane.

Coil Contact Line

Outside the region where the tunnel wall limits a gradient coil, contact between a coil and an adjacent coil of another pair limits the $\tan^{-1}\sqrt{2}$ and $\tan^{-1}\sqrt{8}$ coils. Assuming identical coils in a system, the angle between the gradient axis and the coil contact line is, interestingly enough, the same for the $\tan^{-1}\sqrt{2}$ and $\tan^{-1}\sqrt{8}$ systems and is $\tan^{-1}\frac{1}{\sqrt{2}}$. In the $\tan^{-1}\sqrt{8}$ system, adjacent coils are on alternate sides of the symmetry plane so that all coil contact lines lie in the symmetry plane. In the $\tan^{-1}\sqrt{2}$ system, the coil contact limit surface (conical at $\tan^{-1}\frac{1}{\sqrt{2}}$ with the gradient axis) is also tangent to the symmetry plane. Thus, two pairs of $\tan^{-1}\sqrt{2}$ coils may be placed in one plane to produce the lateral force in that plane, each coil contacting three neighbors, two on the same side of the symmetry plane and one on the other side.

Exclusion Surface

Outside the $\tan^{-1}\sqrt{8}$ coils other coils with their axes parallel to the tunnel axis are to be placed. The geometry involved in determining the space excluded for such coils is seen most easily by noting that

- All points on a circle are equidistant from any point on a line perpendicular to the plane of the circle through its center; and
- The circle lies in a spherical surface of the appropriate radius and whose center lies on that symmetry line.

Thus, the circular turn of a ($\tan^{-1}\sqrt{2}$ or $\tan^{-1}\sqrt{8}$) gradient coil which lies furthermost from the symmetry point determines a spherical surface centered at the symmetry point which provides a safe exclusion volume for the outer coils. These exclusion volumes are indicated in Figures 5 and 6.

The radius of the exclusion cylinder for the main field coils is taken to be the larger of the radius of the ($\tan^{-1}\sqrt{2}$ or $\tan^{-1}\sqrt{8}$) exclusion sphere or the maximum radius of the z-force coil.

COIL CROSS SECTIONS

Generally, the inner boundaries of the various coil cross sections are determined by contact limits or exclusion surfaces. The outer boundaries have been optimized in the following sense. Referring to Figure 7, it can be shown that for given $\rho_1(\phi)$, ϕ_1, ϕ_2 and a constant current density in the coil, the relation between ρ and ϕ which maximizes the z gradient of B at the symmetry point subject to constant joulian power dissipated in the coil is

$$\rho = (\text{Constant})(\sin 2\phi)^{1/3} \quad : \ B_z \text{ - power contour} \ .$$

Similarly, if one asks for maximum B at constant power, the result is

$$\rho = (\text{Constant})(\sin \phi)^{1/2} \quad : \ B \text{ - power contour} \ .$$

The curved outer boundaries of the coil cross sections have been calculated from these relations.

SCALING

It is straightforward to show the manner in which a change of size affects the important magnetic balance parameters. Consider a point in space at which a magnetic field and a magnetic field gradient is being produced by a given configuration of coils with a given resistivity and given constant current density. If one asks how the various quantities change when all distances are multiplied by a scaling factor, ℓ (resistivity and current density unchanged), the results are

∇B	is unchanged
B	is increased by the factor ℓ
Coil volume	is increased by ℓ^3
Coil weight	is increased by ℓ^3
Joulian power	is increased by ℓ^3.

These general results constitute the source of the primary difficulty which arises when one attempts to design a balance for a large wind tunnel. For balances with reasonably large force capabilities using conventional water-cooled copper, coil weight and power quickly get out of hand as the size increases. Thus arises the general conclusion that large balances must utilize more efficient methods of producing magnetic fields; e.g., superconductor and/or supercooled normal conductor coils.

These scaling rules are to be used in conjunction with the design charts of Figures 1, 2, 3, and 4. The various quantities shown on the charts have been calculated for the case where some particular characteristic length (as indicated on each chart) has been chosen to be 1 cm. The scaling rules must be used to get the numbers corresponding to a different size.

In preparing the charts, it has been assumed that the order of placing the coils is as shown in Figures 5 and 6; i.e., the ($\tan^{-1}\sqrt{2}$ or $\tan^{-1}\sqrt{8}$) gradient coils innermost and the z-force and main field coils outside. It seems obvious that for isotropic, large force capacity balances (especially in the larger sizes) this arrangement is the better one. As indicated above, a possible exception is the $\tan^{-1}\sqrt{2}$ system and the possibility of placing z-force and/or main field coils in the space between the tunnel wall and the $\tan^{-1}\sqrt{2}$ coil.

THE USEFUL DESIGN RELATIONS

Force Capacity

Some measure of force capacity is normally a given balance parameter. It is convenient to use the ratio of the maximum balance or support force to the weight of the sphere.

$$\mathrm{wt} = (\mathrm{vol.})\, d_s g$$

$$F = (\mathrm{vol.})\, M f B_z$$

$$\frac{F}{\mathrm{wt}} = \frac{f M B_z}{d_s g} \tag{1}$$

where d_s is the density (gm/cm³) of the sphere, M is the magnetization (or magnetic dipole moment per unit volume) of the sphere and is parallel to the tunnel axis (gauss), g is the acceleration due to gravity (cm/sec²), B_z is $\partial B_z/\partial z$ and z is the gradient coil axis of the pair of gradient coils producing the field gradient (gauss/cm), and f is a nondimensional factor depending on the angle between the gradient coil axis and the magnetization of the sphere ($f = 1$ for zero angle, $1/\sqrt{2}$ for $\tan^{-1}\sqrt{2}$, and $1/\sqrt{3}$ for $\tan^{-1}\sqrt{8}$). In the expression for F, it has been assumed that M and B_z are constant over the volume of the sphere.

Magnetization

It is assumed that the sphere is magnetized by the main field. (Alternately, the sphere could be permanently magnetized and the main field used to keep its magnetic moment oriented parallel to the tunnel axis. This, however, is an undesirable arrangement for dynamic stability studies.) If the main field is larger than that required to saturate the sphere, then M must be the saturation magnetization of the sphere material (this case usually holds for ferrite spheres). For high permeability materials at fields well below saturation, the magnetization of the sphere is the magnitude of the magnetizing field divided by the demagnetizing factor $\left(\frac{4}{3}\pi \text{ for a sphere}\right)$, thus $M = \frac{3B}{4\pi}$. This case holds for soft iron at fields less than about 15,000 gauss.

Power

At least for the initial designs, it is convenient to assume that the coil volumes are completely filled with conductors of uniform and constant resistivity and that the current density (corresponding to maximum force) is uniform. Then, one has

$$\text{Power} = (\text{vol.})\, J^2 \rho$$

where the power (watts) corresponds to the joulian power, the volume is in cm^3, J is the current density ($amps/cm^2$), and ρ is the resistivity (ohm-cm). One notes, of course, that all fields and gradients produced by the coils are directly proportional to the current density and the charts of Figures 1, 2, 3, and 4 correspond to $J = 10^3$ $amps/cm^2$.

DESIGN PROCEDURE

Balances of the configurations indicated in Figures 5 and 6 are designed insideout in the following manner. Initially given (or chosen) information consists of

- Tunnel radius (effective inner radius of coil system)
- Force capacity (F/wt. ratio)
- M and B (from material chosen and mode of operation)
- Current density, J .

1. Equation (1) gives B_z for $\tan^{-1}\sqrt{2}$ or $\tan^{-1}\sqrt{8}$ coil pair.
2. $B_z/2$ = value for one coil ($B_z/4$ if 4 $\tan^{-1}\sqrt{2}$ coils are used for one lateral force).
3. Entering Chart 1 and 2 with appropriate B_z (if J is not 10^3 amps/cm^2 appropriately correct entering B_z), find vol. and D corresponding to R = 1 cm. Scale vol. and D to given tunnel R (B_z is unchanged in scaling).
4. Proceed to z-force coil and Chart 2 in same fashion. The scaling factor here is the exclusion sphere radius D found in the step above.
5. Proceed to main field coil and Chart 4. The scaling factor here is the previous D (for $R_0/K > \sim 0.83$) or the maximum radius of the z-force coil (for $R_0/K < \sim 0.83$). Note that the entering B for Chart 4 must be reduced by the scaling factor since on scaling B is proportional to the scaling factor.

SAMPLE DESIGN

$\tan^{-1}\sqrt{8}$ system for 10 g support. z force (extra z-force gradient coils) for 10 g drag. (F/wt. = 10.) Iron sphere; density = 8 gm/cm^3; B = 5000 gauss (M = 1200 gauss); J = 10^3 amps/cm^2; free tunnel radius = 20 cm (~ 8'').

$\tan^{-1}\sqrt{8}$ Coils

$$\frac{F}{wt} = \frac{fVMB_z}{Vdg} \quad ; \quad f = \frac{1}{\sqrt{3}} \text{ for } \tan^{-1}\sqrt{8} \text{ coils} .$$

two coils: $B_z = \dfrac{10(8)(980)}{(1/\sqrt{3})\,1200} = 113$ gauss/cm .

one coil: $B_z = 56.5$ gauss/cm .

Figure 2

$B_z = 56.5 \rightarrow \beta = 1.445$, volume 3.32 cm^3 .

scaling to 20 cm radius , volume = $3.32(20)^3 = 2.66 \times 10^4$ cm^3 .

radius of exclusion sphere = $\sqrt{3}\,\beta R = \sqrt{3}\,(1.445)20 = 50$ cm .

Water Cooled Copper	Aluminum at 20°K
$\rho = 2 \times 10^{-6}$ ohm-cm; density = 9 gm/cm^3;	$\rho = 1.7 \times 10^{-9}$ ohm-cm; density = 2.7 gm/cm^3
Power = (vol.) $\rho J^2 = 2.66 \times 10^4 (2 \times 10^{-6}) 10^6$ $= 53.2$ kw/coil;	Power $= 2.66 \times 10^4 (1.7 \times 10^{-9}) 10^6$ $= 45$ watts/coil;
wt = vol(d) g $= 2.66 \times 10^4 (9)\ 980$ dynes $= 2.35 \times 10^8$ dynes = 528 #/coil .	wt $= 2.66 \times 10^4 (2.7)\ 980 = 7.05 \times 10^7$ dynes = 158 #/coil .
6 coils:	6 coils:
Total power = 319.2 kw;	Total power = 270 watts;
Total weight = 3168 lb;	Total weight = 948 lb.

z-force Coils

$$\frac{F}{wt} = \frac{fVMB_z}{Vdg} \quad ; \quad f = 1 \text{ for z-force coils} .$$

two coils: $$B_z = \frac{10(8)\ 980}{1200} = 65.3 \text{ gauss/cm} .$$

one coil: $$B_z = 32.7 \text{ gauss/cm} .$$

Figure 3

$B_z = 32.7$ gauss/cm $\rightarrow \frac{R_0}{K} = 0.90$, volume $= 0.27\ \text{cm}^3$.

scaling by exclusion sphere radius above = 50 cm ,

volume $= 0.27(50)^3 = 3.38 \times 10^4\ \text{cm}^3$.

radius of exclusion cylinder = 50 cm (since $R_0/K > 0.83$) .

Water Cooled Copper	Aluminum at 20°K
Power $= 3.38 \times 10^4(2 \times 10^{-6})10^6$ watts $= 67.6$ kw/coil;	Power $= 3.38 \times 10^4(1.7 \times 10^{-9})10^6$ $= 67.5$ watts/coil;
wt $= 3.38 \times 10^4(9)(980) = 2.98 \times 10^8$ dynes = 670 #/coil;	wt $= 3.38 \times 10^4(2.7)\ 980 = 8.95 \times 10^7$ dynes = 201 #/coil;
two coils:	two coils:
Total power = 135.2 kw; Total weight = 1340 lb.	Total power = 115 watts; Total weight = 400 lb.

Main Field Coils

B = 5000 for two coils .

B = 2500 for one coil .

Scale by radius of exclusion cylinder = 50 cm .

Figure 4

$$\ell = 0 \; , \quad B = \frac{2500}{50} = 50 \rightarrow \gamma = 1.24 \; , \quad \text{volume} = 0.64 \text{ cm}^3 \; .$$

scaling by radius of exclusion cylinder = 50 cm ,

$$\text{volume} = 0.64(50)^3 = 8 \times 10^4 \text{ cm}^3 \; .$$

maximum radius = $\gamma(50)$ = 62 cm .

Water Cooled Copper	Aluminum at 20°K
Power = $8 \times 10^4(2 \times 10^{-6})10^6$ = 160 kw/coil;	Power = $8 \times 10^4(1.7 \times 10^{-9})10^6$ = 136 watts/coil
Weight = $8 \times 10^4(9)\ 980 = 7.05 \times 10^8$ dynes	Weight = $8 \times 10^4(2.7)\ 980$ dynes
= 1585 #/coil ;	= 476 #/coil ;
two coils:	two coils:
Total power = 320 kw;	Total power = 272 watts;
Total weight = 3170 lb.	Total weight = 952 lb.

Grand Total

Water Cooled Copper

Power = 319.2 kw + 135.2 kw + 320 kw = 774 kw;

Weight = 3168 lb. + 1340 lb. + 3170 lb. = 7678 lb.

Aluminum at 20°K

Power = 270 watts + 115 watts + 272 watts = 657 watts;

Weight = 948 lb. + 400 lb. + 952 lb. = 2300 lb.

Figure 1 B_z and volume vs. D for one $\tan^{-1}\sqrt{2}$ gradient coil. X section boundaries are B_z power contour, tunnel wall contact line, and (if $D > 3$) coil contact line. Numbers correspond to tunnel radius $R = 1$ cm and current density $J = 10^3$ amps/cm^2. D = radius of exclusion sphere.

Figure 2 B_z and volume vs. β for one $\tan^{-1}\sqrt{8}$ gradient coil. X section boundaries are B_z power contour, tunnel wall contact line, and coil contact line. Numbers are for tunnel radius $R = 1$ cm, and current density $J = 10^3$ amps/cm^2. $D = \sqrt{3}\ \beta$ is radius of exclusion sphere.

Figure 3 B_z and volume vs. R_0/K for one z-force gradient coil. X section boundaries are B_z power contour and circle of radius R_0 about symmetry point. Numbers are for $R_0 = 1$ cm and current density $J = 10^3$ amps/cm^2. Radius of safe exclusion sphere is K. (Radius of exclusion cylinder is 1 cm for $1/K > \sim 0.83$.)

Figure 4 B and volume vs. γ for one main field coil. X section boundaries are B power contour, line parallel to tunnel axis at radius 1 cm, and a line perpendicular to tunnel axis ℓ cm above symmetry point. (Extended B power contour intersects plane through symmetry point perpendicular to tunnel axis γ cm from tunnel axis.) Current density $J = 10^3$ amps/cm^2. When coil is scaled by a linear factor, B is increased by that factor.

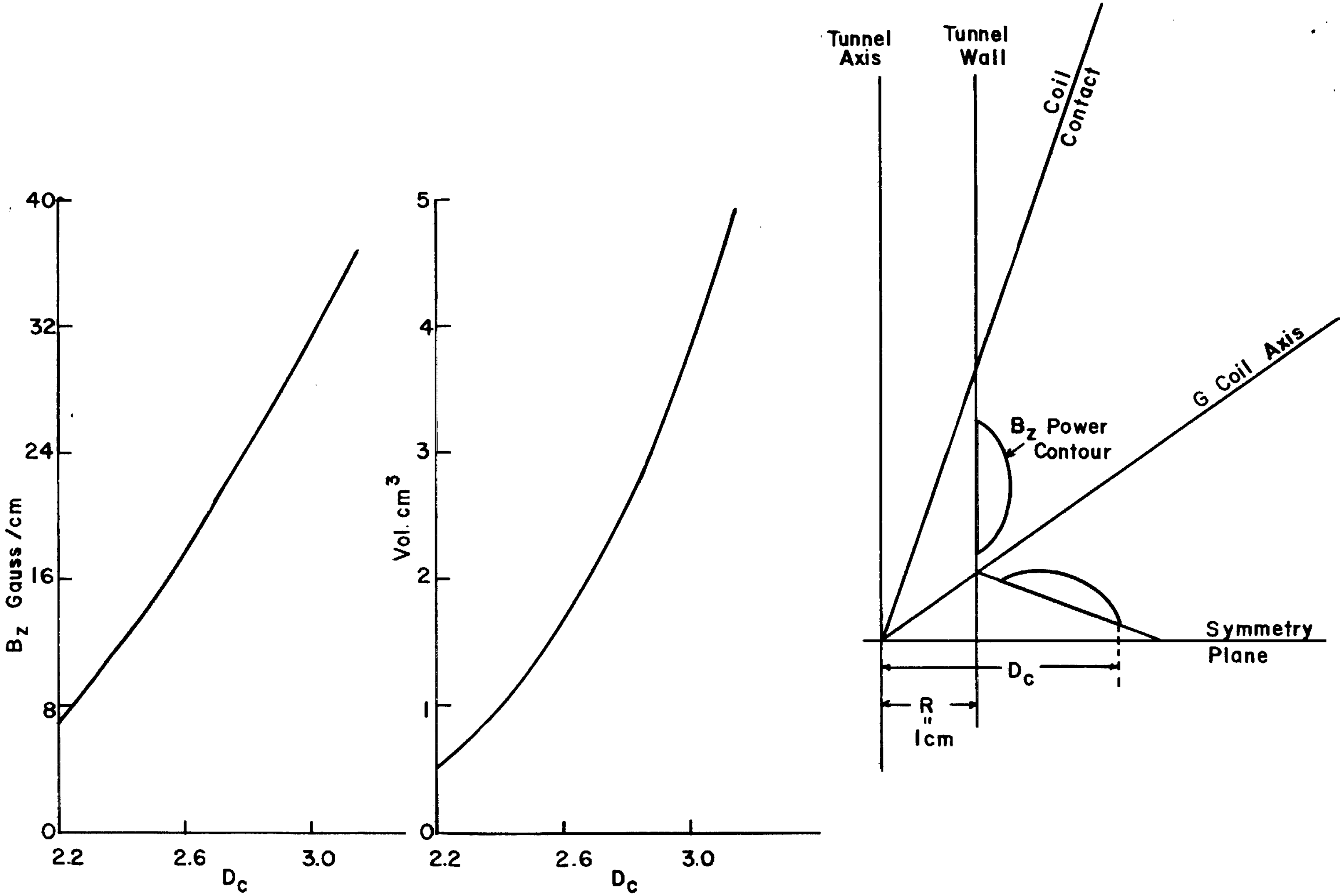

FIGURE 1
ONE $\tan^{-1}\sqrt{2}$ COIL

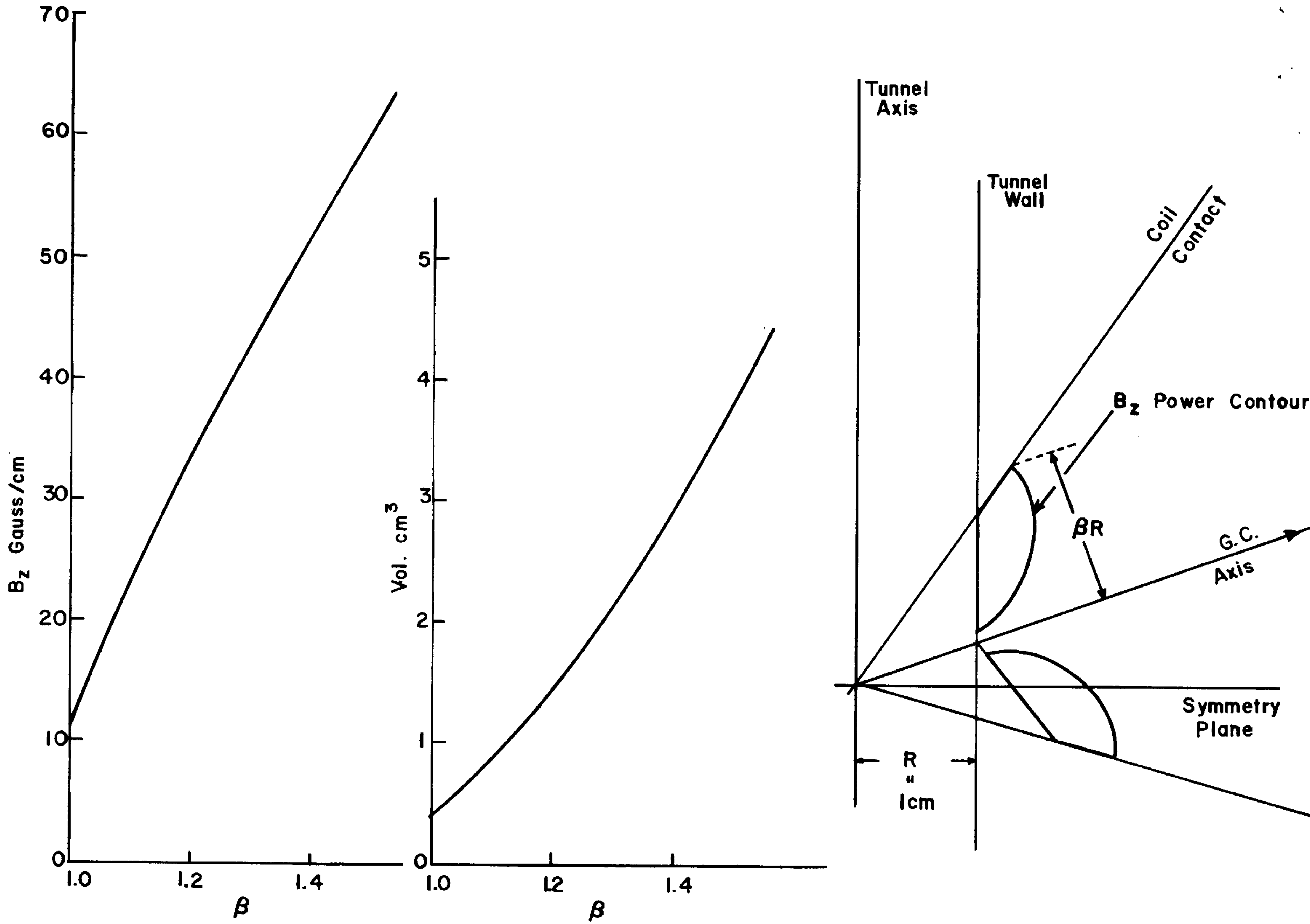

FIGURE 2
ONE $TAN^{-1}\sqrt{8}$ COIL (R=1cm)

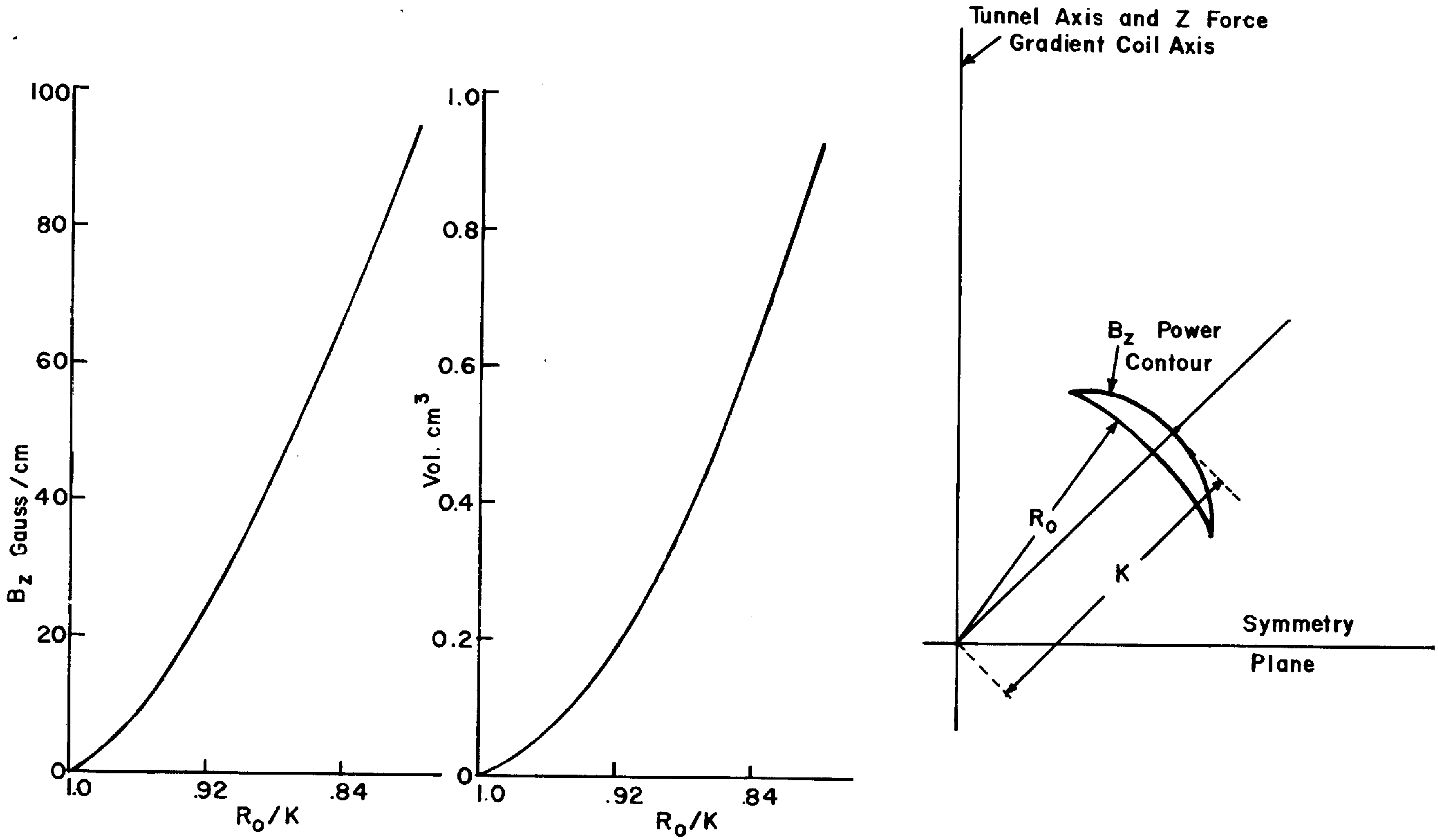

FIGURE 3
ONE Z FORCE COIL (R_o = 1 cm)

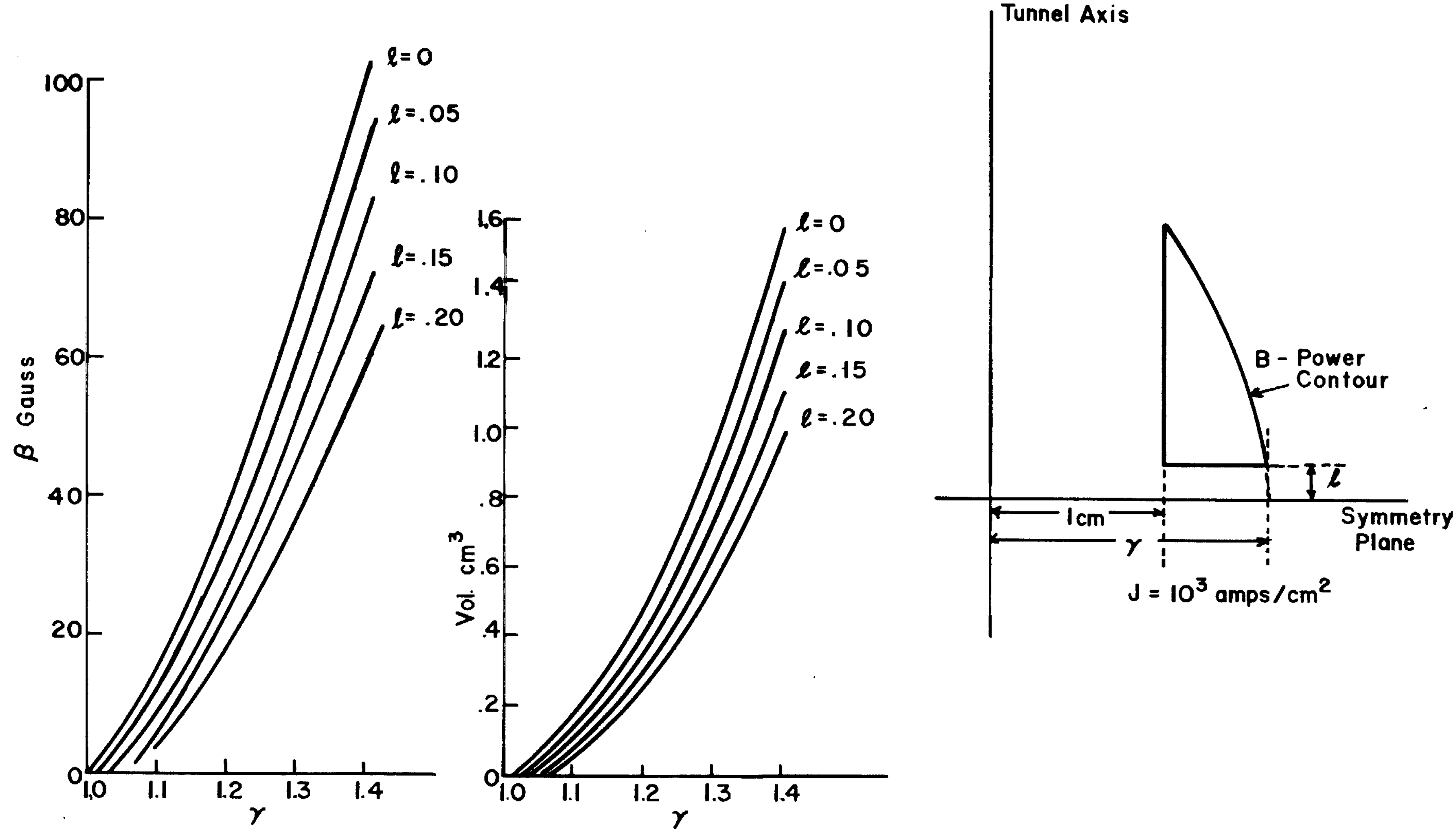

FIGURE 4
ONE MAIN FIELD COIL

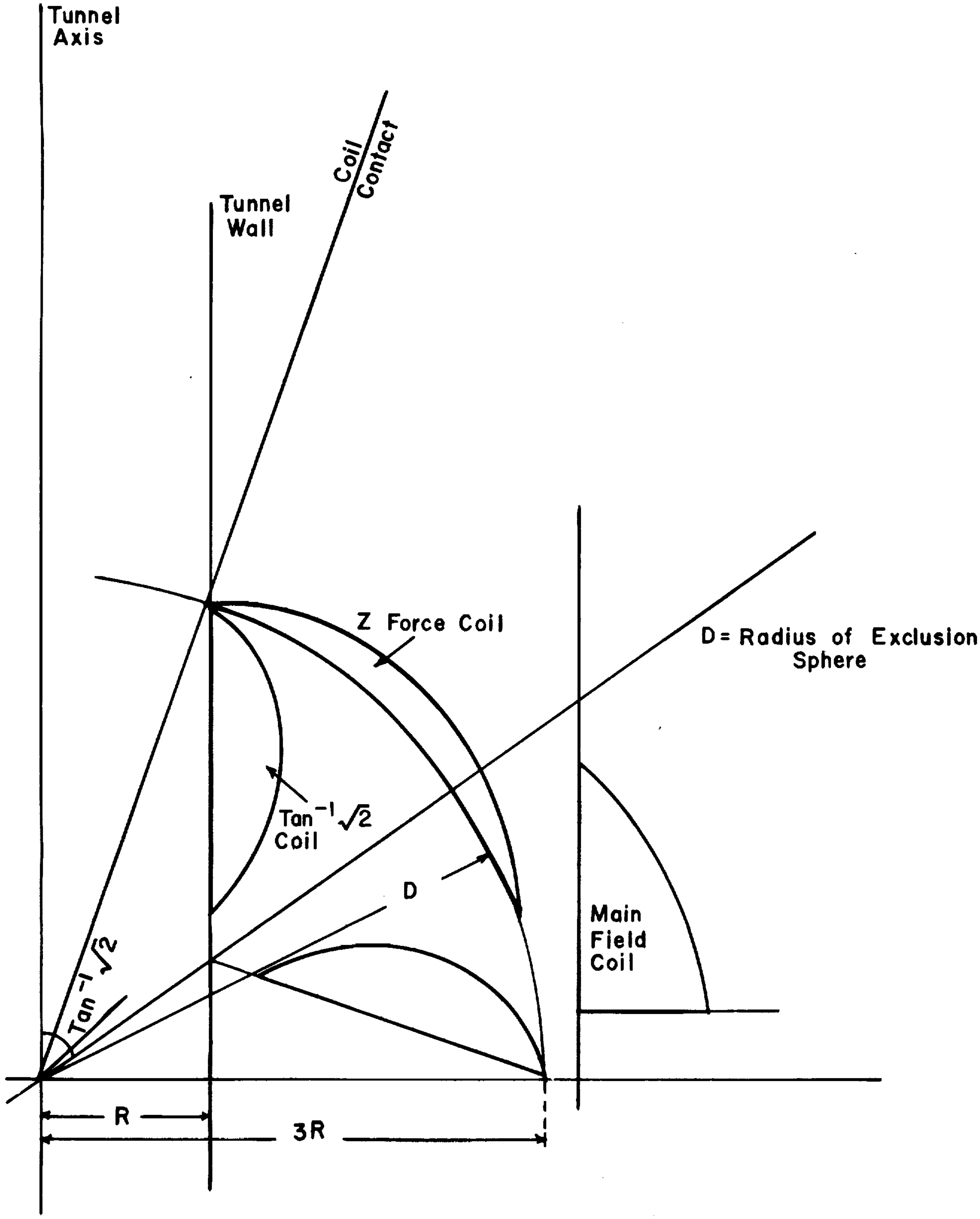

FIGURE 5

A TYPICAL $TAN^{-1}\sqrt{2}$ BALANCE CONFIGURATION

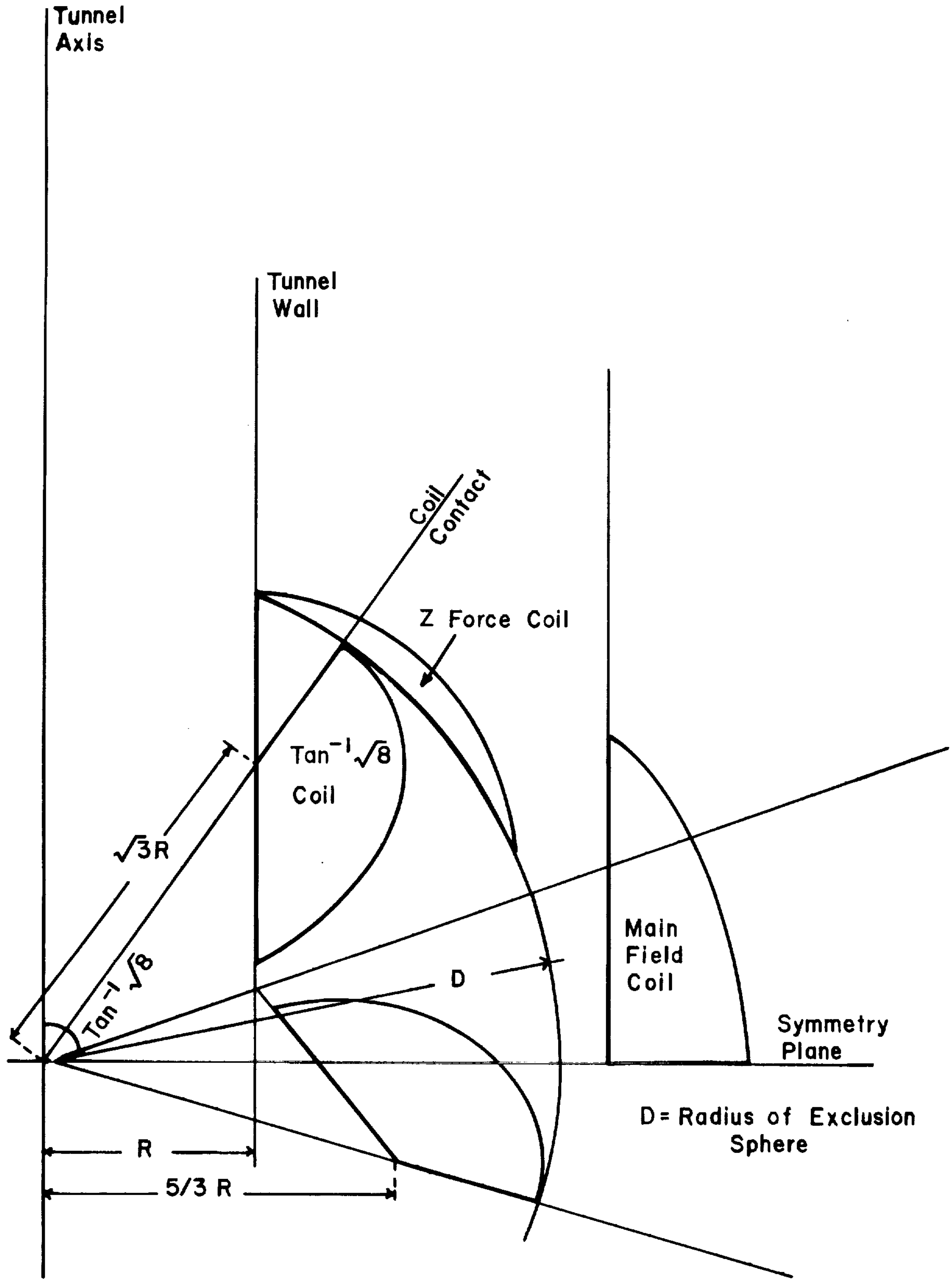

FIGURE 6
A TYPICAL TAN$^{-1}\sqrt{8}$ (DRAG AUGMENTED) BALANCE CONFIGURATION

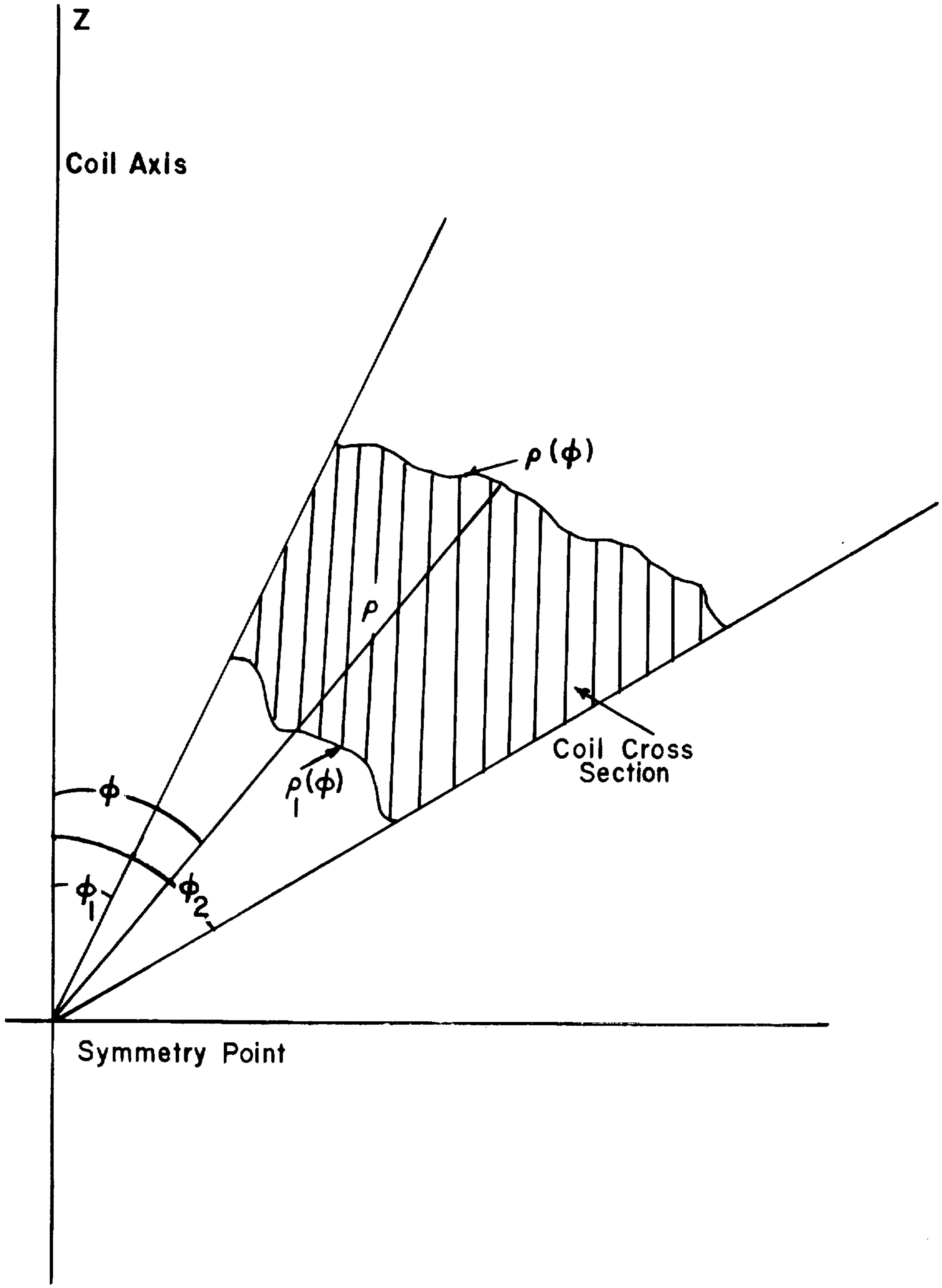
Z
Coil Axis
$\rho(\phi)$
ρ
Coil Cross Section
$\rho_1(\phi)$
ϕ
ϕ_1
ϕ_2
Symmetry Point

FIGURE 7

DISTRIBUTION LIST

Copy No.	
1 - 25	Grants and Research Contracts Division Office of Space Science and Applications Code SC National Aeronautics and Space Administration Washington, D. C. 20546
26 - 28	H. M. Parker
29	A. R. Kuhlthau
30	H. S. Morton
31 - 35	RLES Files